## **Chapter 6 Linear and Ideal Transformers**

**6.1 Mutual Inductance**

* Figure 6.1.1 shows two coils in air, wound on a former made from nonmagnetic material.

* Let the current  in coil 1 be time-varying, whereas coil 2 is open circuited. A voltage  is induced in coil 1, in accordance with Faraday’s law:

  (6.1.1)

where *φ*1e is an effective flux of coil 1 associated with *i*1, which if it links all *N*1 turns gives  Thus, . *φ*1e accounts for the fact that in the case of cores of low permeability, not all of the magnetic flux links all the turns of the coil (Figure 6.1.1).

* Let  be the fraction of the time-varying flux  that links coil 2, where is an effective flux that if multiplied by *N*2 gives the flux linkage *λ*21 in coil 2 due to *i*1. A voltage  is induced in this coil in accordance with Faraday’s law:

  (6.1.2)

where .

* The quantity  is defined as the flux linking coil 2 per unit current in coil 1. Thus:

  (6.1.3)

* If a time-varying current  is applied to coil 2, with coil 1 open circuited, then following the same argument, we have, analogous to Equations 6.1.1 to 6.1.3:

  (6.1.4)

  (6.1.5)

  (6.1.6)

where  is the flux linking coil 1 per unit current  in coil 2.

* We will show that *M*12 = *M*21 by determining the energy expended in establishing steady currents  and  starting from zero. It is convenient to assume that  and  are established in two steps: i)  is first increased from zero to  with ; and ii)  is then increased from zero to  with 
* For the sense of winding of coil 1 in Figure 6.1.1, the flux associated with  is downward in this coil, according to the right-hand rule. While  is increasing, the induced voltage  =  in coil 1 opposes the increase in  in accordance with Lenz’s law, by being a voltage drop across *L*1 in the direction of *i*1. This voltage is concurrently a voltage rise across the current source, so that the total energy  delivered by the source is:

  (6.1.7)

* With  (Figure 6.1.2), the voltage induced in coil 2 is that due to increasing  and the total energy supplied by the current source *iSRC*2 in establishing  is  as in Equation 6.1.7.
* As  increases it induces a voltage  in coil 1. The sense of winding of coil 2 and the direction of  are such that the flux associated with  is also downward in coil 1. The effect of increasing  is therefore the same as that of increasing  so that  is of the same polarity as  in Figure 6.1.1, and opposes the current in coil 1. The current source *iSRC*1 has therefore to deliver additional energy to maintain :

 **** (6.1.8)

* The total energy expended in establishing  and  is:

  (6.1.9)

* If  and  are established in the reverse order, then following the same argument as above, the total energy expended in establishing  and  is:

  (6.1.10)

*  = , because in a lossless, linear system, the total energy expended must depend only on the final values of  and  and not on the time course of  and . Otherwise, it would be possible, at least in principle, to extract energy from the system at no energy cost, in violation of conservation of energy.
* It follows that:

  (6.1.11)

* *M* is the **mutual inductance** between the two coils and is a constant in linear systems. In contrast, the individual inductances *L*1 and *L*2 are **self-inductances**.

***Definition*** *The mutual inductance of two magnetically-coupled coils is the flux linkage in one coil per unit current in the other coil. It is independent of which coil carries the current.*

* If either the polarity of *iSRC*2, or the sense of winding of coil 2, is reversed in Figure 6.1.2, the flux due to  becomes upward in coil 1. The polarity of  is reversed and becomes a voltage drop across the current source *iSRC*1. Energy is therefore returned to the source and the sign of the energy term involving  becomes negative in Equations 6.1.9 and 6.1.10. *M*, however, is *always* a positive quantity.
* Since  and  are arbitrary values, they might just as well be replaced by instantaneous values and  The energy stored in the magnetic field in building up the currents in two magnetically coupled coils to *i*1 and *i*2, starting from zero, may therefore be expressed in general as:

  (6.1.12)

**Coupling Through a High-Permeability Core**

* If coils 1 and 2 are wound on a toroidal core of high, constant permeability (Figure 6.1.3), then because of the shape and high permeability of the core, the flux in the core follows a circular path that is *confined entirely to the core and links all the turns of both coils*. In other words, the effective flux that links all the turns of either coil is the same as the actual flux.
* The flux that links only one coil is the **leakage** **flux**. It is in the air space between the coils and the core and generally does not link all the turns of the coil.
* If *i*1 is the current in coil 1, with *i*2 = 0, the total flux linkage *λ*1 of coil 1 may be expressed as:

 *λ*1 = *N*1*φ*11e + *N*1*φ*21 (6.1.13)

where *N*1*φ*21 is the flux linkage due to the flux *φ*21 in the core, and *N*1*φ*11e is the flux linkage due to the leakage flux. The self-inductance  of coil 1 is:

 (6.1.14)

* Replacing  in Equation 6.1.3 by *M*, and  by *φ*21, the mutual inductance becomes:

 (6.1.15)

* If *i*2 is the current in coil 2, with *i*1 = 0, the relations corresponding to Equations 6.1.13 to 6.1.15 are obtained by repeating the above arguments. Thus:

 *λ*2 = *N*2*φ*22e + *N*2*φ*12 (6.1.16)

 (6.1.17)

 (6.1.18)

**Coupling Coefficient**

* Multiplying together Equations 6.1.15 and 6.1.18:

  (6.1.19)

 Dividing by the product *L*1*L*2 from Equations 6.1.14 and 6.1.17:

  (6.1.20)

* The expression  is a measure of how effectively coil 1 is coupled to the core. Similarly for . The product on the RHS of Equation 6.1.20 is a measure of how well the two coils are magnetically coupled together through the core.
* Equation 6.1.20 may be written as:

  or  (6.1.21)

***Definition*** *The coupling coefficient k of two magnetically-coupled coils is defined as*  *and is a measure of how tightly the two coils are coupled through the core. It assumes values in the range of 0 to unity, where k = 0 denotes no coupling and k = 1 denotes perfect coupling.*

|  |
| --- |
| **Table 6.1.1 Electric Circuit Analogy** |
| **Electric Circuit** | **Magnetic System** |
| Current | Flux (*φ*) |
| Voltage excitation (emf) | mmf (*Ni*) |
| Resistance | Reluctance (mmf/flux) |
| Conductance  | Permeance (flux/mmf) |
| Conductivity | Permeability |

**Electric Circuit Analogy**

* A useful analogy can be made between a magnetic systems and electric circuits (Table 6.1.1).
* Magnetic flux is analogous to electric current, and magnetomotive force (mmf), which equals *Ni* and drives magnetic flux, is analogous to voltage excitation, also known as electromotive force (emf). Flux is the product of

permeance and mmf.

**6.2 The Linear Transformer**

***Definition*** *A transformer consists of two or more coils that are magnetically coupled relatively tightly. In a linear transformer, permeability is constant, so that B and H, or φ and i, are linearly related.*

* In transformer terminology, the coil connected to the source of excitation is the **primary winding**, whereas the coil connected to the load is the **secondary winding** (Figure 6.2.1).

* Because of linearity, the flux in the core is the superposition of *φ*21, associated with *i*1 alone, and *φ*12, associated with *i*2 alone.
* KVL for the primary circuit is: , where *R*1 is the resistance of coil1. From Equation 6.1.14, , and from Equation 6.1.18: . Substituting:

  (6.2.1)

* KVL for the secondary circuit is: . Substituting From Equations 6.1.15 and 6.1.17:

  (6.2.2)

**Sign of Mutual Inductance Term**

* The sign of the mutual inductance term depends on the relative sense of the windings. Instead of having to show the sense of the windings, a dot convention is used.

***Dot Convention*** *One terminal of each coil is marked with a dot so that currents entering, or leaving, the marked terminals in each coil are associated with flux in the same direction in both coils.*

* In Figure 6.2.1, for example, we may arbitrarily place a dot on terminal 1 of coil 1. Since *i*1 entering at this terminal is associated with flux in the core in the clockwise direction, and *i*2 entering terminal 2 is also associated with flux in the same direction, terminal 2 is dotted. Alternatively, terminals 1′ and 2′ may be dotted. If the sense of winding of either coil is reversed, as in Figure 6.1.3, then the dotted terminals will be 1 and 2′, or 1′ and 2. In Figure 6.1.2, terminals 1 and 2, or 1′ and 2′, will be dotted.
* An alternative interpretation of the dot markings, which follows from the above, is that *the polarities of induced voltages in both coils are the same, relative to the dot markings*. In Figure 6.1.2, *w*hen *i*2 is increasing, *v*2 opposes the increase in *i*2 and *v*12 opposes *I*1. Both voltages oppose currents entering at the dotted terminals, and the polarities of these voltages make the dotted terminal positive with respect to the unmarked terminal in both coils.
* Once the terminals are marked with dots, the sign of the *M* term readily follows:

***Sign of M Term*** *If the assigned positive directions of currents are such that these currents both flow in, or both flow out, at the dotted terminals, the sign of the mutual inductance term (Mdi1*/*dt, or Mdi2* /*dt) for either coil is the same as that of the self-inductance term for that coil (L1di1*/*dt, or L2di2* /*dt) . Otherwise, the sign of the mutual inductance term for either coil is opposite that of the self-inductance term for that coil.*

* The justification is that if the assigned positive directions of coil currents are such that both currents flow into, or out of, the dotted terminals,  and  produce flux in the same direction in the core. This means that the  voltage induced by  in coil 1 is of the same polarity as the  voltage induced by  in coil 1, so that these two terms have the same sign in the voltage relations of coil 1. Similarly for the voltage induced by  in coil 2 and the  term.

**Frequency-Domain Representation**

* Equations 6.2.1 and 6.2.2 are expressed in the frequency domain by replacing the time-varying currents with the corresponding phasors and replacing differentiation by *jω*. (Figure 6.2.2). Thus:

 **I1** – *jωM***I2** = **VSRC** (6.2.3)

and, -*jωM***I1** + **I2** = 0 (6.2.4)

**T-Equivalent Circuit**

* Equations 6.2.3 and 6.2.4 are satisfied by the T-equivalent circuit of Figure 6.2.3.

* If the dot markings on either coil are reversed, the sign of *M* is reversed (Figure 6.2.4).

**Example 6.2.1 Mesh-Current Analysis of Circuit Including Coupled Coils**

 Given the circuit of Figure 6.2.5 in which *vSRC* = 100cos800*t* and *k* = 0.25. It is required to determine the steady-state value of *vO*.

***Solution*:** *ωL*1 Ω; *ωL*2  Ω; *M* = = 5×10-3 H; *ωM* Ω; and  Ω. The circuit in the frequency domain is shown in Figure 6.2.6.

In writing the mesh current equation for mesh 1, the total voltage drop in this mesh due to **I1** equals, as usual, **I1** multiplied by the total self-impedance of this mesh, that is,(10 + *j*8 – *j*5)**I1**. **I2** introduces as usual a voltage rise of *Zc***I2** in mesh 1, where *Zc* =-*j*5 Ω is the common impedance between meshes 1 and 2, plus a *jωM***I2** term due to the magnetic coupling between the coils in the two meshes. Since both **I1** and **I2** enter at the dotted terminals, the sign of the *jωM***I2** term is the same as that of *jωL*1 term in the equation of mesh 1 and the same as that of the *jωL*2 term in the equation for mesh 2. The mesh-current equation for mesh 1 is therefore:

  (6.2.5)

 Similarly, the mesh-current equation for mesh 2 is:

  (6.2.6)

Solving for **I2** gives: **I2** = -3.0636 – *j*0.5375 A, so that **VO** = 5**I**2 = -15.3 – *j*2.69 = 15.55∠-170.0° V, or *vO* = 15.55cos(800*t* – 170.0°) V.

 Equations 6.2.5 and 6.2.6 could just as well be derived using the T-equivalent circuit (Figure

6.2.7). If *L*1 in Figure 6.2.6a is rotated clockwise and *L*2 is rotated counterclockwise so as to bring them to the upright position, the dot markings will be as in the transformer of Figure 6.2.4b. The appropriate T-equivalent circuit is therefore that having series branches *L*1 + *M* and *L*2 + *M* and a shunt branch –*M*.

 If the assigned positive direction of **I2** in Figure 6.2.4 is made counterclockwise, and this current is denoted by  then  flows into the unmarked terminal of coil 2. The sign of the mutual inductance term is now opposite that of the self-inductance term for each coil. The *Zc* term in the mutual impedance between the two meshes becomes positive since the voltage drop due to flowing in *Zc* is also a voltage drop in mesh 1. Equations 6.2.5 and 6.2.6 become:

  (6.2.7)

and,  (6.2.8)

 Now = –**I2** and **Vo** = –*R*2= *R*2**I2** as before. The T-equivalent circuit is the same as in Figure 6.4.6b.

 If the dot marking on coil 2 is reversed, with the mesh current **I2** still counterclockwise, the sign of the mutual inductance term is again the same as that of the self-inductance terms. Equations 6.2.7 and 6.2.8 become:

  (6.2.9)

and,  (6.2.10)

 When *L*1 and *L*2 are rotated so as to bring them to the upright position, it is seen that dot markings are those of the transformer of Figure 6.2.4a. Using the corresponding T-equivalent circuit gives the same Equations 6.2.9 and 6.2.10.

 Under dc conditions, the flux does not vary with time and no voltage is induced in either coil due to current in the coil itself or due to current in the other coil. The inductances behave as short circuits. Moreover, capacitor acts as an open circuit. If *VSR°C* = 100 V, it follows from voltage division that **VO** =  V.

**6.3 The Ideal Transformer**

***Concept*** *When a time-varying voltage v is applied to a coil, then neglecting the coil resistance, flux linkage λ is established in the coil in accordance with Faraday’s law v = dλ*/*dt, irrespective of the parameters of the coil and of the characteristics of the medium in which the magnetic flux flows. On the other hand*, *the coil current is determined by the inductance of the coil, which in turn depends on the coil and on the characteristics of the medium in which the magnetic flux flows.*

* In Fig. 6.3.1, a sinusoidalvoltage *v*1 is impressed across coil 1, the primary winding of a transformer, with coil 2, the secondary winding, open circuited.

* *L*1 has two components: (i) a component *Lc* due to the magnetic path in the core, and (ii) a component *L*leak due to the leakage path. Since the emfs induced in these paths are in series, the two components of *L*1 are also in series.
* Let the relative permeability of the core *μr* become infinite. Since inductance increases with permeability, the coil inductance *L*c, and hence *L*1 also becomes infinite, which means that , because a coil of infinite inductance, and hence infinite impedance, draws no current.
* If , the mmf acting on the leakage path is zero, so the leakage flux is zero, since flux = permeance×mmf. The induced voltage in the leakage path is also zero, so that *v*1 appears across *L*c.
* The mmf acting on the core is also zero, but because the core is assumed to be of infinite permeability, and hence of infinite permeance, the flux *φc* in the core is indeterminate from the relation flux = permeance×mmf, but is in fact finite, as required by Faraday’s law, which takes the form, *v*1 = *N*1*dφc*/*dt*.
* If , then *p* = *v*1*i*1 = 0, so no work is done in establishing *φc*, and hence no magnetic energy is stored in the core. In other words, no work is done in

establishing a finite flux in a core of infinite permeability.

* *φc* induces a voltage *v*2 in coil 2 such that *v*2 = *N*2*dφc*/*dt*. Dividing *v*1 = *N*1*dφc*/*dt* by *v*2:

  (6.3.1)

* Equation 6.3.1 may be expressed as: , which emphasizes that since *φc* is

common to both coils, the volts per turn are the same for both coils.

* If *i*2 ≠ 0 and *μr* is finite, *φc* is related to *i*1 and *i*2 through Ampere’s circuital law, which now takes the form: , because the magnetic field due to *i*2 opposes that due to *i*1. Multiplying the RHS of this equation by  gives *Bc*, and multiplying by the cross-sectional area *A* gives *φc*:

  (6.3.3)

* If , then in order that *φc* remains finite, (*N*1*i*1 – *N*2*i*2) = 0. The physical interpretation is that if the permeability of the core is very high, then only a negligibly small net magnetomotive force (mmf), (*N*1*i*1 – *N*2*i*2), is associated with a finite *φc*.
* This is exactly analogous to a finite current flowing through a connection of very high conductance, the voltage across the conductance being negligible.
	+ With (*N*1*i*1 – *N*2*i*2) = 0, it follows that:

  (6.3.4)

* With *i*1 and *i*2 flowing, Equation 6.3.1 no longer strictly applies, because there will now be a voltage drop due to coil resistance and there will be an mmf acting on the leakage path to produce a leakage flux and hence a voltage drop. Thus, in order that both Equations 6.3.1 and 6.3.4 apply, we have to assume negligible resistance and perfect coupling between the coils and the core.

* If the assigned positive

direction of  and  do not conform to the dot markings . Similarly, if  and  both enter or leave at the dotted terminals,  so that . The four possibilities are illustrated in Figure 6.3.2.

* It follows from the voltage and current ratios that the instantaneous power input equals the instantaneous power output, neglecting power losses in the core.

***Definition*** *An ideal transformer is a two-port device that neither dissipates nor stores energy, and whose input-output v-i relations are of the form:  and *

* From Equation 1.9.8:  and , so that:

  (6.3.5)

independently of *μr*. Thus, although each of *L*1 and *L*2 becomes infinite as *μr* becomes infinite, their ratio remains finite and equal to the square of the turns ratio.

**Phasor Relations**

* Figure 6.3.3b shows the phasor diagram for the ideal transformer of Figure 6.3.3a. Figure 6.3.3c is a flow diagram of the causal relationships.
* A voltage **V1** applied to the primary winding establishes a flux **φc** in the core such that **φc** **V1** and lags **V1** by 90°. **φc** induces a voltage **V2** in the secondary winding such that **V2****φc**, assuming the assigned positive directions and dot markings shown. Since **V2** leads **φc** by 90°, it is in phase with **V1**. **V2** causes a current **I2** in the

secondary circuit, which lags **V2** by an angle *θ*, assuming *ZL* is inductive. In order to have zero mmf in the core, a current **I1** flows in the primary winding such that **I1****I2**.

**Reflected Impedance**

* For the dot markings and assigned positive directions of Figure 6.3.3a:  and . Dividing these two equations: . Substituting :

  (6.3.6)

* Because of squaring of the turns ratio, Equation 6.3.6 is valid for any of the configurations of Figure 6.3.3a to d.
* An open circuit is reflected as an open circuit, and a short circuit is reflected as a short circuit.

**Example 6.3.1 Ideal Transformer Circuit**

 Assume that in Figure 6.3.4, *v*1 = 120cos1000*t*, *N*1:*N*2 = 1:2, and that *Z* consists of a resistance of 10 Ω in series with a 20 mH inductor. It is required to determine the primary current and the secondary current and voltage.

***Solution*:** **V2** = **V1** = 240∠0° V; hence **I2** =

∠-63.4° A. It follows that **I1** = **I2** ∠-63.4° A.

**The Ideal Autotransformer**

* Consider a two-winding transformer having  (Figure 6.3.5a). *v*1 can be stepped up to *v*2 by adding to *v*1 a voltage , resulting in a step-up autotransformer (Figure 6.3.5b).
* A winding of  is required, and the current in the common winding is *i*1 – *i*2, which means that this winding can have a conductor of smaller cross-sectional area. Both of these considerations make the autotransformer smaller and lighter. In addition, terminal a in Figure 6.3.5b could be connected to a slider over a bare part of the winding, so that a variable turns ratio is obtained.

* The principal disadvantage of the autotransformer is that the input and output sides are not electrically isolated, because of the conduction path between them.
* The voltage and current relations for the autotransformer follow directly from those of the two-winding transformer by substituting . Thus:

  (6.3.8)

  (6.3.9)

* If the dot marking on either winding is reversed, the effective turns ratio becomes .

**Example 6.3.2 Three-Winding Transformer**

 Given a three-winding ideal transformer with loads *Z*2 and *Z*3 connected as shown in Figure 6.3.7. It is required to determine the input impedance.

***Solution*:** *Method 1*: Since the voltage induced per turn is the same for all windings, , and

, where the assigned positive directions of **V1**, **V2**, and **V3** are all in accordance with the dots, so that the volts per turn are positive for all the windings. Since the net mmf in the core must be zero,

= 0, where  and . Note that **I1** and (**I1** – **I2**) enter at the dotted terminals of their respective windings. Their mmfs are in the same sense and may be assigned a positive sign. **I3** leaves its winding at the dotted terminals, so its mmf is assigned a negative sign.

 Eliminating , , , and  from these equations gives:

  (6.3.10)

 If , the impedance reflected to the primary side is , as for a two-winding transformer of turns ratio . If , the reflected impedance is , as for an autotransformer of turns *N*1 and *N*2. In fact, these reflected impedances in parallel give the reflected impedance of Equation 6.3.10.

*Method 2*: Let the impedances be replaced by current sources **I2** and **I3**, in accordance with the substitution theorem, as shown in Figure 6.3.8, where  and . We now apply superposition. If  is applied alone, the primary current is . If  is applied alone, the primary current is . With both sources applied, the primary current is **I1** =  + . Substituting for the currents in terms of voltages and impedances: **I1** =  + . But = **V1** and =**V1**. Substituting for  and  gives: **I1** = **V1**. The reflected impedance **V1**/ **I1** is the same as that obtained above.

 Note that superposition strictly applies to voltage sources and current sources only. It cannot be applied directly to impedances, such as *Z*2 and *Z*3 in this Example. However, the substitution theorem allowed us to apply superposition after replacing these impedances with sources.

**6.4 Reflection of Circuits**

***Concept*** *Circuits involving ideal transformers can be conveniently analyzed by reflecting the circuit on the primary side to the secondary side, or conversely.*

* Consider Figure 6.4.1. KCL for node q is:

**I2** + **IL**(**V2** – **Vy**)

  (6.4.1)

* Substituting: **V2** = *a***V1** and **I2****I1**, Equation 6.4.1 may be expressed as:

**I1****IL** (6.4.2)

where .

* This is KCL for a node q′ in Figure 6.4.2. The voltage **V1** at node p and the current **I1** leaving this node, are unaltered. The ideal transformer now appears at the extreme right with its secondary open-circuited. Since it is not serving any useful function, it can be omitted from the circuit.

* In Figure 6.4.2, the secondary circuit is reflected to the primary side, element by element. If *a* > 1, voltages are *stepped down* by *a* and currents are *stepped* *up* by *a*. Impedances are stepped down, like voltages, but by a factor *a2.*
* In a similar manner, we may write KCL for node p in Figure 6.4.1 as:

  (6.4.3)

* Substituting:  and , Equation 6.4.3 may be expressed as:

  (6.4.4)

where . This is KCL for a node p′ on the secondary side in Figure 6.4.3. The voltage and currents at node q are unaltered. The ideal transformer now appears at the extreme left of the figure with its primary open-circuited. Since it is not serving any useful function, it can be omitted from the circuit.

* In reflecting circuits from one side to the other, the order of the elements, left to right or right to left, must be preserved. Otherwise, the circuit is altered.
* Controlling currents or voltages of dependent sources are reflected like any other voltage or current.
* When the dot markings are reversed, a negative value of *a* is used.

**6.5 Transformer Imperfections**

* Practical transformers depart from the ideal in the following respects:
* finite inductance of the windings;
* finite leakage flux;
* power losses, because of finite resistance of the windings and core losses;
* capacitances between primary and secondary windings as well as between layers of the same winding. The capacitances arise because of voltage

differences and consequent energy storage in the electric field.

* The finite resistances of the windings are accounted for, at least at low frequencies, by adding them at the terminals of the respective winding
* The distributed capacitance between windings and layers of the same winding are accounted for in an approximate manner by lumped capacitances connected across the terminals of the primary and secondary windings and between these windings.
* Ignoring the aforementioned imperfections, except for finite inductances and imperfect coupling, we are left with a linear transformer consisting of two coupled, lossless coils, of inductances *L*1 and *L*2 (Figure 6.5.1).
* The governing equations are Equations 6.2.3 and 6.2.4, with *R*1 = 0 = *R*2, and with *RL* replaced by *ZL*:

**I1** – *jωM***I2** = **VSRC** (6.5.1)

-*jωM***I1** + **I2** = 0 (6.5.2)

* Any circuit that represents the effect of finite inductances of windings and finite leakage fluxes must satisfy these equations.

**Finite Inductance of Windings**

* Let the coils to be perfectly coupled to begin with, that is, . Substituting for **I2** from Equation 6.5.2 in Equation 6.5.1:

 (6.5.3)

* With , it follows that the impedance seen by the source is that of *jωL*2 in parallel with *ZL*, reflected to the primary side of an ideal transformer having an inductance ratio of *L*1:*L*2, or a turns ratio  (Equation 6.3.5), as illustrated in Figure 6.5.2a. The impedance *jωL*2 reflected to the primary side becomes *jωL*1 (Figure 6.5.2b).

* It is seen that the effect of finite inductances of an otherwise ideal transformer is to introduce a shunt impedance *jωL*1 on the primary side, or a shunt impedance *jωL*2 on the secondary side.

**Finite Leakage flux**

* Because equivalence must apply for any value of *ZL*, we may assume that Z*L* = 0. Substituting  in Equation 6.5.3:

  (6.5.4)

* When terminals 22′ in Figure 6.5.2b are short circuited by having *ZL* = 0, *jωL*1 is short circuited. To satisfy Equation 6.5.4, a series impedance must be inserted in series at terminal 1, as shown in Figure 6.5.3.
* When terminals 22′ are open circuited, **I2** = 0, and Equation 6.5.1 gives: . To satisfy this condition, the shunt impedance must be instead of , as

shown. Moreover, *L*2 in Figure 6.5.2a must now be reflected as *k*2*L*1 in order to satisfy this same condition, which implies that the inductance ratio is *k*2*L*1:*L*2.

***Concept*** *The performance of a transformer is limited at low frequencies by the reactance of the windings and at high frequencies by the leakage reactance.*

* At low frequencies, the leakage impedance is negligible, but the impedance *jωL*2 appears in parallel with *ZL*. If the transformer is not to affect adversely the behavior of the circuit, *jωL*2 >> *ZL*.
* At high frequencies, the shunting effect of *jωL*2 can be neglected. The leakage impedance referred to the secondary side becomes  and appears in series with the load. If the transformer is not to affect adversely the

behavior of the circuit, this impedance should be small compared with *ZL*.

**Core Losses**

* These are of two types: eddy-current losses and hysteresis losses.
* Eddy-current losses occur in any core made of electrically conducting material. Suppose that the secondary in Figure 6.3.1 consists of a single, closed turn of wire of resistance *R*. A voltage of rms value is induced in this turn and causes a current , assuming the inductance of the turn is negligible with respect to its resistance  The  power loss is /*R*.

* But a similar situation occurs throughout the body of the core, since any closed path inside the core may be considered to act like the closed turn of wire, as illustrated in Figure 6.5.4a.
* This causes **eddy currents** to circulate in the core. Apart from the power loss and the resultant heating, the flux of these eddy currents (*φe* (*t*) in Figure 6.5.4a opposes, and hence decreases, the flux in the core.
* To reduce these currents, either a magnetic material of high resistivity is used, or the core is made of thin laminations that are insulated from one another and stacked together so that the flux is in a direction parallel to the plane of the laminations (Figure 6.5.4b). The induced currents are confined by the insulation to within each lamination. This effectively reduces the cross sectional area of the loop that encloses flux and which can give rise to current flow.
* Iron, steel, nickel, cobalt and their alloys are **ferromagnetic** materials, characterized by high permeability and nonlinearity that includes **hysteresis**. In general, hysteresis arises when an effect lags behind its cause. As a result, the state of a system depends on its previous history, that is, the manner in which this state was reached.
* In the case of ferromagnetic materials, if  in an unmagnetized specimen is increased from zero,  increases along the curve OP in Figure 6.5.5.
* The flattening of the curve at high values of  is described as **magnetic saturation**.

* If at point P,  is reduced,  lags behind. When  is reduced to zero at Q, *B* retains the value of the positive intercept on the *B*-axis. To reduce *B* to zero at R requires a negative *H*. Making *H* more negative still brings the operating point to S*.* If  is now increased back to *Hm*,  changes along the lower part of the curve, STUP.
* The loop that is traced by a cyclic variation in  is a hysteresis loop. The area enclosed by the hysteresis loop represents power loss that appears as heat in the core. It is seen that at any a particular value *Hx*, for example,  can take on different values corresponding to points 1, 2, or 3, or intermediate values, depending on how *Hx* is reached.
* Both eddy-current loss and hysteresis loss are a function of the magnitude of *φc*.